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Supersymmetry and higher derivative terms in the effective action of Yang-Mills theories

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Abstract: Higher derivative terms in the effective action of certain Yang-Mills theories can be severely constrained by supersymmetry. We show that requiring sixteen supersymmetries in quantum mechanical gauge theory determines the v^6 term in the effective action. Even the numerical coefficient of the v^6 term is fixed in terms of lower derivative terms in the effective action.

Keywords: M(atrix) Theories, Extended Supersymmetry, Supersymmetric Effective Theories.

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Contents

1. Introduction

A better understanding of Yang-Mills theories with extended supersymmetry is crucial if we are to gain a deeper understanding of the various non-perturbative field theory dualities. For example, extended supersymmetry plays a key role in making possible conjectures about exact strong-weak coupling dualities in four-dimensional theories with eight and sixteen supersymmetries [1]. Yang-Mills theories with extended supersymmetry have also played a prominent role in a recent attempt to define M theory [2]. In that endeavor, the theory of interest is the quantum mechanical gauge theory that describes the low-energy dynamics of zero-branes in type IIA string theory [3, 4]. The system can be obtained by a dimensional reduction of supersymmetric Yang-Mills from ten dimensions [5]. The theory has sixteen supersymmetries and a $U(N)$ gauge symmetry. For finite N, this matrix model is believed to describe M theory quantized in the discrete light-cone formalism (DLCQ) [6, 7].

More generally, we should ask the question: to what extent does supersymmetry determine the form of the effective action of Yang-Mills theories? In a recent paper, we proved a non-renormalization theorem for the v^4 term in the effective action of D0-brane quantum mechanics [8]. The aim of this letter is to apply the same technique to the v^6 term to show that it is also determined by supersymmetry. Quantum mechanical gauge theory with sixteen supersymmetries is a quite subtle theory. Since the coupling has positive mass dimension, the theory is strongly coupled at low energies. For example, in matrix theory $g^2 = M_{pl}^6 {R_{||}}^3$ where $R_{||}$ is the size of the longitudinal direction and g^2 is the Yang-Mills coupling.

More importantly, the theory has a highly non-trivial vacuum for any N as conjectured in [4] and proven for $N = 2$ in [9]. Studying an effective action obtained by perturbing around the trivial vacuum is unlikely to make much sense at higher orders in a derivative expansion. It seems much like trying to analyze the long wavelength physics of QCD using perturbation theory. Indeed, recent arguments suggest that at order v^8, the perturbative derivative expansion breaks down [10]. What should be surprising is that perturbative computations actually gave results that agreed with supergravity for

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the v^4 and v^6 terms [11, 12, 13].[1] As we shall see, the reason for such agreement is essentially the strong constraints imposed by supersymmetry on the effective action. It seems likely that the construction of a complete effective action will first require developing a somewhat new perturbation theory for scattering amplitudes. A correct perturbation theory, along the lines described in [9], must encorporate the non-trivial vacuum structure. This is a fascinating problem that will be explored elsewhere. That supersymmetry determines the F^4 and F^6 terms in ten-dimensional Yang-Mills has been shown in [16]. Comments on the general structure of Yang-Mills effective actions have appeared in [17, 18, 19].

2. Constraining the six derivative terms

Ignoring acceleration terms, the bosonic part of the D0-brane effective action takes the form

$$S = \int dt \, \left(f_1(r)v^2 + f_2(r)v^4 + f_3(r)v^6 \ldots \right). \tag{2.1}$$

A discussion of the Lagrangian for four-dimensional Yang-Mills including acceleration terms is given in [20]. For the most part, we shall restrict our discussion to the effective action describing the dynamics of two clusters of D0-branes. The Lagrangian contains both bosonic fields x^i as well as fermions ψ_a, where $i = 1, \ldots, 9$ and $a = 1, \ldots, 16$.

The $Spin(9)$ Clifford algebra can be represented by real symmetric matrices γ^i_{ab}, where $i = 1, \ldots, 9$ and $a = 1, \ldots, 16$. These matrices satisfy the relation,

$$\{\gamma^i, \gamma^j\} = 2\delta^{ij}, \tag{2.2}$$

and a complete basis contains $\left\{I, \gamma^i, \gamma^{ij}, \gamma^{ijk}, \gamma^{ijkl}\right\}$, where we define

$$\begin{aligned} \gamma^{ij} &= \frac{1}{2!}(\gamma^i\gamma^j - \gamma^j\gamma^i) \\ \gamma^{ijk} &= \frac{1}{3!}(\gamma^i\gamma^j\gamma^k - \gamma^j\gamma^i\gamma^k + \ldots) \\ \gamma^{ijkl} &= \frac{1}{4!}(\gamma^i\gamma^j\gamma^k\gamma^l - \gamma^j\gamma^i\gamma^k\gamma^l + \ldots). \end{aligned} \tag{2.3}$$

The basis splits into symmetric, $\left\{I, \gamma^i, \gamma^{ijkl}\right\}$, and antisymmetric matrices, $\left\{\gamma^{ij}, \gamma^{ijk}\right\}$. The normalizations in (2.4) are chosen so that the trace of the square of a basis element is ± 16.

Supersymmetry demands that f_1 be constant and $f_2 = \frac{c_2}{r^7}$ [8]. We will choose $f_1 = \frac{1}{2}$. The coefficient c_2 is determined by a one-loop computation [11]. The Lagrangian L can be expressed as the sum of terms, $L = \sum L_k$, where L_k contains all terms of order $2k$. For example,

$$L_1 = \int dt \left(\frac{1}{2} v^2 + i \, \psi\dot{\psi} \right). \tag{2.4}$$

[1] Comments about a puzzle [14] for the v^6 terms in higher rank theories have recently appeared in [15].

The order counts the number of time derivatives plus twice the number of fermions. Schematically at order 6, we need to consider all terms,

$$L_3 = \int dt \left(f_3^{(0)}(r)\, v^6 + \ldots + f_3^{(12)}(r)\, \psi^{12} \right), \tag{2.5}$$

which are in the supersymmetric completion of v^6. The omitted terms contain accelerations and fermions with multiple time derivatives. The supersymmetry transformations take the general form:

$$\begin{aligned} \delta x^i &= -i\epsilon\gamma^i\psi + \epsilon N^i\psi \\ \delta\psi_a &= (\gamma^i v^i \epsilon)_a + (M\epsilon)_a \,. \end{aligned} \tag{2.6}$$

The terms N^i and M encode all higher derivative corrections to the supersymmetry transformations and ϵ is a sixteen component Grassmann parameter. Note that once higher derivative terms appear in L, we must have N^i and M non-zero or the supersymmetry algebra no longer closes. The actual construction of N^i and M is a tedious business. Fortunately, as in the case of the v^4 term, we will not need to know very much about N^i and M to show that the v^6 term is also determined by supersymmetry.

The terms in L_2 generate corrections to the supersymmetry transformations of order 2 in N^i and of order 3 in M. These corrections are fully determined by L_2. When we include the six derivative terms in L_3, we get higher derivative terms in N^i of order 4 and in M of order 5. We will only need to know the order of the terms in N^i and M.

We primarily wish to consider the twelve fermion term which is the 'top' form in the supersymmetric completion of v^6. A study of the analogous term in the completion of v^4 gave a non-renormalization theorem for the v^4 term.[2] The variation of this term in (2.5) schematically contains two pieces,

$$\delta(f_3^{(12)}(r)\psi^{12}) = \delta f_3^{(12)}(r)\ \psi^{12} + f_3^{(12)}(r)\delta\psi^{12} \,. \tag{2.7}$$

Acting on terms with order 6, we need only consider the lowest order free-particle supersymmetry transformations. The variation of L_3 then gives terms of order 6, where we count ϵ as order $-1/2$. The first term in (2.7) contains a thirteen fermion term. Note that no other term in L_3 varies into the thirteen term. Can any term from L_1 vary into a thirteen fermion term? The highest-order term in N^i is order 4, which can contain an eight fermion term. The highest term in M can contain a ten fermion term. It is easy to check that the variation of L_1 given in (2.4) cannot then contain a thirteen fermion term.

We can ask the same question about terms from L_2. The top form in L_2 is an eight fermion term which is non-vanishing and shown in [8] to agree with the form computed at one-loop in [21]. The relevant term in N^i is order 2 and so can contain a four fermion term, while the relevant term in M is order 3 and so can contain a six fermion term.

[2]It is worth stressing that essentially the same argument used to determine the four derivative terms in [8] can be applied in any dimension to four derivative terms in Yang-Mills theories with only eight supersymmetries and a flat metric.

Therefore, a variation of the top form in L_2 can generate a thirteen fermion term. These are the only two sources of thirteen fermion terms in the Lagrangian.

The last piece of information that we need is the number of independent twelve fermion terms. These terms need to be invariant under the discrete symmetry which acts as complex conjugation and sends,

$$x \to -x \qquad\qquad t \to -t.$$

All n fermion structures $T_{a_1 \ldots a_n}$ are Hodge dual to $16-n$ fermion structures using the epsilon symbol in sixteen dimensions. We therefore only need to ask how many independent four fermion structures are possible. It is easy to check using the Fierz identities in Appendix A of [8] that the only allowed independent structure is

$$x^i x^j \left(\psi \gamma^{ik} \psi \psi \gamma^{kj} \psi \right). \tag{2.8}$$

Therefore, there is a unique twelve fermion structure,

$$T_{a_1 \ldots a_{12}} = \epsilon_{a_1 \ldots a_{12} b_1 b_2 b_3 b_4} \left(x^i x^j \gamma^{ik}_{b_1 b_2} \gamma^{kj}_{b_3 b_4} \right), \tag{2.9}$$

and we define

$$T = T_{a_1 \ldots a_{12}} \psi_{a_1} \cdots \psi_{a_{12}}.$$

There are two possible cases: either the terms from L_2 make a contribution to the thirteen fermion term in the variation of L, or they do not. Let us assume they do not make a contribution. This implies that

$$\begin{aligned} \delta_a \left(f_3^{(12)} T \right) &= -i \gamma^s_{ab} \psi_b \, \partial_s \left(f_3^{(12)} T \right) \\ &= 0 \,. \end{aligned} \tag{2.10}$$

We can apply the operator $\gamma^q_{ac} \frac{d}{d\psi_c} \partial_q$ to (2.10). After summing over the a index, we learn that,

$$\Delta \left(f_3^{(12)} T \right) = 0,$$

which gives the equation

$$\frac{d^2}{dr^2} f_3^{(12)} + 12 \frac{1}{r} \frac{d}{dr} f_3^{(12)} = 0 \,. \tag{2.11}$$

However, the solution to this equation $f_3^{(12)} \sim 1/r^{11}$ is unphysical since it implies that the twelve fermion term is proportional to a negative power of the coupling. A tree level twelve fermion term would need a power of $1/r^{14}$. Actually, we should note that eq. (2.11) is weaker than eq. (2.10), and one can show directly from (2.10) that the function $f_3^{(12)}$ vanishes.

We should now consider the case where the terms from L_2 do contribute. The terms in L_2 are one-loop exact so power counting is easy. The eight fermion term is proportional to $\frac{1}{r^{11}}$. The relevant corrections to the supersymmetry transformations have the following dependence on r:

$$N^i \sim \frac{1}{r^9}\psi^4$$
$$M \sim \frac{1}{r^{10}}\psi^6\,,$$

in accord with the one-loop exactness of L_2. Equation (2.11) then becomes

$$\frac{d^2}{dr^2}f_3^{(12)} + 12\frac{1}{r}\frac{d}{dr}f_3^{(12)}\,, + \frac{c_2'}{r^{24}} = 0, \tag{2.12}$$

where the non-zero coefficient c_2' is determined by the terms in L_2. As we have seen, the homogeneous solution to (2.12) where $c_2' = 0$ is unphysical so we discard it. The remaining solution gives

$$f_3^{(12)} = -\frac{c_2'}{242}\frac{1}{r^{22}}\,, \tag{2.13}$$

which is precisely the power needed to agree with a two-loop calculation like the one performed in [11]. Therefore, the v^6 terms are also not renormalized but are completely determined by supersymmetry and the lower derivative terms in the effective action. The same method can be used to learn about the six derivative terms in Yang-Mills theories with sixteen supersymmetries in various dimensions.

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